NAG Toolbox for MATLAB

s14af

1 Purpose

s14af returns the value of the kth derivative of the psi function $\psi(z)$ for complex z and $k = 0, 1, \ldots, 4$, via the function name.

2 Syntax

```
[result, ifail] = s14af(z, k)
```

3 Description

s14af evaluates an approximation to the kth derivative of the psi function $\psi(z)$ given by

$$\psi^{(k)}(z) = \frac{d^k}{dz^k}\psi(z) = \frac{d^k}{dz^k}\left(\frac{d}{dz}\log_e\Gamma(z)\right),$$

where $z = x + iy$ is complex provided $y \neq 0$ and $k = 0, 1, \ldots, 4$. If $y = 0$, z is real and thus $\psi^{(k)}(z)$ is singular when $z = 0, -1, -2, \ldots$.

Note that $\psi^{(k)}(z)$ is also known as the *polygamma* function. Specifically, $\psi^{(0)}(z)$ is often referred to as the *digamma* function and $\psi^{(1)}(z)$ as the *trigamma* function in the literature. Further details can be found in Abramowitz and Stegun 1972.

s14af is based on a modification of the method proposed by Kölbig 1972.

To obtain the value of $\psi^{(k)}(z)$ when z is real, s14ae can be used.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Kölbig K S 1972 Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

5 Parameters

5.1 Compulsory Input Parameters

1: **z – complex scalar**

The argument z of the function.

Constraint: Re(**z**) must not be 'too close' (see Section 6) to a nonpositive integer when Im(**z**) = 0.0.

2: **k – int32 scalar**

The function $\psi^{(k)}(z)$ to be evaluated.

Constraint: $0 \leq \mathbf{k} \leq 4$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – complex scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{k} < 0$,
or $\mathbf{k} > 4$,
or $\mathrm{Re}(\mathbf{z})$ is 'too close' to a nonpositive integer when $\mathrm{Im}(\mathbf{z}) = 0.0$. That is, $\mathrm{ABS}(\mathrm{Re}(\mathbf{z}) - \mathrm{NINT}(\mathrm{Re}(\mathbf{z}))) <$ ***machine precision*** $\times \mathrm{NINT}(\mathrm{ABS}(\mathrm{Re}(\mathbf{z})))$.

ifail $= 2$

The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

7 Accuracy

Empirical tests have shown that the maximum relative error is a loss of approximately two decimal places of precision.

8 Further Comments

None.

9 Example

```
z = complex(-1.5, +2.5);
k = int32(1);
[result, ifail] = s14af(z, k)
```

```
result =
  -0.1974 - 0.2427i
ifail =
           0
```